

ADVANCED EXAMPLES IN PHYSICS

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NOTE

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I BELIEVE every University teacher of Physics will welcome Mr Allen's book of examples. At various times and in a spasmodic fashion I have thought of writing something of the kind myself, but foreseeing little glory and much toil I eschewed the task, although for the past twenty years I have wanted a work of this kind.

Answers are appended to the problems. This is the only feature of the book I rather regret, but they are inserted in the interests of private pupils. I should be delighted if only some of the answers were *wrong* so that students (whose notion of working examples is to juggle with the numerics in a question so as to get the numeric in the answer) might be righteously confounded, but I have no such hope; for Mr Allen's carefulness and exactitude are such that in his preface he does not even tell the users of his book that "any corrections to the answers will be gratefully received."

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AUTHOR'S PREFACE.

CONSIDERABLE difficulty has been met with in the Degree courses in Physics at Leeds University in setting numerical examples of a fairly advanced type, and I doubt not that the same difficulty will have been felt elsewhere. No teacher can afford to dispense with so important a test of the progress of his students as the working of calculations based on the subject-matter of the lectures; yet there appears to be no collection of suitable exercises published at a moderate price. The dictation of questions to a class absorbs valuable time, and the handing round of loose leaflets is a troublesome process. Moreover the large number of students working privately for the examinations of London University must find themselves at a great disadvantage if, as often happens, they have not easy access to a set of University Calendars. This book is designed to meet the difficulty referred to; most of the questions are taken from old examination-papers, the source being acknowledged by a letter in brackets at the end of each exercise. The London and Victoria Universities are indicated by the letters L. and V. respectively; H. denotes that the paper was one for Honours candidates. When no letter is added, the question is either original or has been taken from the lecture note-books in use at Leeds; for permission to use the latter I am greatly indebted to Professor Stroud. Questions taken from old papers have as a rule been reproduced in their original form even when improvement seemed possible; the detection of such cases is in itself a useful exercise.

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GENERAL PHYSICS.

1. Find the number of watts in one horse-power, given
1 foot = 30.48 cms.; 1 lb. = 453.6 gms.; $g = 981$ cms. per sec.
per sec. (L. 1902.)

2. A body oscillates under a restoring force proportional to the displacement. Shew that slight frictional resistance steadily diminishes the amplitude, but does not sensibly affect the period. Shew by dimensions or otherwise that if the restoring force varies as the n th power of the displacement, the period varies inversely as the $\frac{1}{2}(n-1)$ th power of the amplitude. (L. 1898.)

3. Justify on theoretical grounds the empirical law that the resistance of a medium devoid of rigidity to the passage of a sphere varies as the density of the medium and the square of the velocity and diameter jointly. Define the "terminal velocity." (L. 1901.)

4. Shew that if a particle be subject to an impressed force g per unit mass, and to a resistance per unit mass equal to k times the velocity, then however quickly or slowly the particle be started, it will eventually move at a certain constant speed in the direction of the impressed force. Determine the magnitude of this "terminal velocity."

5. On the experimental law that the resistance of similar steamers varies as the area of the wetted surface and the square of the velocity, prove that if a 6 ft. model of 0.01 ton displacement run at a speed of 2 knots in an experimental tank experiences a resistance of 0.2 lb., a similar 600 ft. 10,000 ton steamer at 20 knots would experience a resistance equivalent to an incline of 1 in 112, and require over 12,000 effective horse-power. (L. 1902.)

6. Assuming the experimental laws that the normal pressure of the wind and the tangential friction of the water per sq. ft. are proportional to the square of the velocity, prove that a ship and its model are heeled over to the same angle by a wind proportional to the square root of the length or the sixth root of the tonnage and move through the water at a proportional speed. (L. 1903.)

7. On the above experimental laws, prove that steamers geometrically similar, propelled at speed proportional to the sixth root of the tonnage, will experience the same resistance in lbs. per ton and will burn the same coal per ton-mile; but the H.P. per ton and the period of revolution of the screw will be proportional to the speed, and the steam pressure to the square of the speed. (L. 1903.)

8. Taking the H.P. per ton as $\frac{1}{18}$ of the speed in knots, prove that this implies a resistance of about 20 lbs. per ton, or an equivalent incline of 1 in 112; and with a coal consumption of 2 lbs. per H.P. hour, the coal capacity required for a voyage of 3000 mls. is about a sixth of the tonnage. Calculate for a steamer of 26,000 tons at a speed of 23.5 knots. (L. 1903.)

9. The mean density of the earth is 5.5; find the attraction between two small spheres a metre apart, each weighing one kilogram, if the earth's radius is 6.37×10^8 cms. (V. H. 1892.)

10. If the force between two spheres, masses M_1 and M_2 , d cms. apart, is GM_1M_2/d^2 where $G = 6.66 \times 10^{-8}$, find the time of revolution of a dust particle round a metal sphere of density 10, when no other bodies act on the system and the dust particle just clears the sphere. (L. H. 1895.)

11. A particle moves uniformly round a circle with angular velocity ω , shew that its acceleration is towards the centre and equal to $\omega^2 r$. Assuming the Law of Gravitation, and taking the orbits of the Earth round the Sun and of the Moon round the Earth as circular, compare the masses of the Sun and of the Earth, given that the Moon makes 13 revolutions per year, and that the Sun is 390 times as distant as the Moon. (L. 1894.)

12. A train of weight 200 tons can be drawn by an engine at the uniform speed of 30 mls. an hr. up an incline of 1 in 200, or at 40 mls. an hr. up an incline of 1 in 400. Assuming the frictional resistance to be constant, calculate it in lbs. per ton; also the H.P. exerted by the engine, assuming it to be the same in both cases. (L. 1898.)

13. A stream of water flowing at the rate of 1100 lbs. per sec. turns an undershot water-wheel. It comes against the paddles with a velocity of 8 ft. per sec., and leaves with the velocity of the paddles, 4 ft. per sec. Find the rate of loss of momentum and the rate of working of the wheel. (L. 1896.)

14. A bullet fired horizontally from a rifle pierces in succession three thin screens placed at intervals of 1000 ft.; and the times by chronometer of passing from the 1st screen to the 2nd, and from the 2nd to the 3rd, are found to be .847 sec. and .948 sec. respectively. Find the ratio of the retarding force (assumed to be uniform) to the weight of the bullet. (L. 1891.)

15. A hoop weighing 2 lbs. and 2 ft. in diameter is rolling at the rate of 30 revolutions per min. along a horizontal road. Find the kinetic energy of the hoop in ft. lbs. (The plane of the hoop is vertical, and the point of it in contact with the ground momentarily at rest.) (L. 1895.)

16. A heavy particle falls down a rough inclined plane which makes an angle of 45° with the horizon; the velocity of the particle after falling down the plane is half the velocity which would have been gained by the particle if it had fallen freely through the vertical height of the plane. Find the coefficient of friction between the particle and the plane. (L. 1893.)

17. A ballistic pendulum whose time of complete vibration is 4 seconds, has a bob whose mass is 100 kilograms. A shot weighing 30 grams fired into the bob produces an oscillation of the pendulum of 1° on each side of the zero. Calculate the velocity of the shot. (V. 1903.)

18. A solid cylinder 8 inches in diameter and weighing 5 lbs. rolls down a rough plane 10 ft. long and falling 1 in 16. Find the time taken, and the final angular velocity of the cylinder. (V. 1901.)

19. How would you determine the moment of inertia of the pulley in an Atwood's machine? If the rim of this pulley were 10 cms. in diameter and the moment of inertia were 1500 c.g.s. units, what would be the acceleration of a system in which the weights are 200 and 250 gms. respectively at a place where $g = 981$ cms. per sec. per sec.? (V. 1891.)

20. A wheel and axle of given dimensions in fixed bearings is started from rest by a weight attached to a flexible cord wrapped round the axle. Shew how to calculate the acceleration if the bearings be smooth, and how to estimate the friction if they are rough by an observation of the time taken for a given length to unwrap. (L. 1892.)

21. A straight uniform rod of length α and mass m hinged at one end is released from a horizontal position; calculate the angular velocity of the rod when it passes through the vertical position. (V. 1905.)

22. A thin uniform heavy rod of length l and mass m , pivoted freely at one end, is released from the horizontal position. If the kinetic energy is divided into two parts, one of rotation about the centre of gravity and the other of translation of the centre of gravity, calculate the amount of each type. (V. 1903.)

23. A moveable mass is attached to a compound seconds pendulum; determine the positions in which the time will be (i) less than, (ii) equal to, and (iii) greater than, a second. (V. 1903.)

24. A body performs torsional oscillations about a fixed axis; obtain an expression for its angular velocity in any position. Shew that its potential energy at any time is equal to $\frac{2I\pi^2\alpha^2}{T^2}$ where I is the moment of inertia, α the angular displacement, and T the time of oscillation. (V. 1902.)

25. A uniform wire of length 3α is bent into the form of an equilateral triangle. Calculate the length of the simple equivalent pendulum (i) for oscillations about a vertex in the plane of the triangle, and (ii) for similar oscillations in the perpendicular plane. (V. 1905.)

26. Find the centre of oscillation of a square plate whose edge is α oscillating under gravity with its plane always vertical, when the point of suspension is (i) at a corner, (ii) at the middle of one side. (V. 1900.)

27. A horizontal circular disc of radius α is suspended by a wire; calculate by how much the time of torsional oscillation

will be altered, if the point of attachment, instead of being at the centre of the disc, lies at a distance $\frac{a}{10}$ from the centre, the disc being kept horizontal by a small weight attached to the rim equal to $\frac{1}{3}$ of the weight of the disc. (V. 1899.)

28. A sphere of mass 98 gms. and radius 3 cms. is suspended by a wire of mass 2 gms. and length 50 cms. from a knife-edge of small mass. Find the length of the equivalent simple pendulum. (V. 1898.)

29. A thin heavy hoop of diameter D is supported by a peg at its circumference and makes small oscillations in its own plane. Calculate the time of an oscillation. (V. 1896.)

30. An inertia-bar, supported by a metal wire, vibrates torsionally. Compare the times of vibration of this system with a similar system in which (i) the linear dimensions of the inertia-bar are all doubled, (ii) the linear dimensions of the wire are all doubled, (iii) the linear dimensions of both bar and wire are all doubled. (V. 1903.)

31. A vertical steel wire carries an inertia-bar at its lower end and the time of torsional vibration is 2.7 secs. If the linear dimensions of the wire are all doubled and those of the bar are halved, the materials being unchanged, find the new period of vibration. (V. 1896.)

32. Given that Young's Modulus for steel is 2×10^{12} in c.g.s. units, calculate its value in pounds' weight per square inch, given that 1 ft. = 30 cms., and 1 kilogram = 2.2 lbs. ($g = 981$ cms. per sec. per sec.) (V. 1905.)

33. A wire 10 metres long and 0.01 sq. cm. in cross-section is stretched 0.1 cm. by a load of 2.5 kilograms. Find the value of Young's Modulus in c.g.s. units. (L. 1893.)

34. A horizontal beam, clamped at one end and free at the other, is loaded at the free end and the sag produced there is measured. How would the sag be affected by doubling the linear dimensions of the beam, the load remaining the same?

(V. 1900.)

35. If a cylindrical beam 150 cms. long and of cross-sectional area 2 square cms. be loaded in the middle with 10 kilograms, calculate the sag if Young's Modulus for the material is 2×10^{12} c.g.s. units.

(V. 1903.)

36. A hollow metal tube 1 cm. in internal and 1.4 cm. in external diameter, whose length is 80 cms., is supported at the ends and loaded with 5 kilograms in the middle. Calculate the sag if Young's Modulus for the metal is 2×10^{12} c.g.s. units. ($g = 981$ cms. per sec. per sec.)

(V. 1905.)

37. A uniform wire 3 metres long and 2 mms. thick can be twisted 2.5 times round without permanent set. If the moment of the force it then exerts be equal to that of a weight of 200 gms. at an arm of 20 cms., calculate the average energy in ergs per c.cm. stored in the twisted wire.

(V. 1894.)

38. If a load of 2 kilograms at the end of a lever 30 cms. long, applied to one end of a circular rod $\frac{1}{8}$ cm. in diameter and $\frac{1}{2}$ m. long, twist its point of application through 10° , the other end being firmly clamped, what is the rigidity of the rod in c.g.s. units?

(L. H. 1891.)

39. Prove that $\frac{1}{Y} = \frac{1}{9k} + \frac{1}{3n}$, where Y , k , and n are respectively Young's Modulus and the bulk and rigidity moduli for a substance. If this equation is used to find k for a metal of which Y is about 2×10^{12} and n is about 10^{12} , these quantities being determined with errors of the order of one per cent., what may be the error in k ?

(L. H. 1902.)

40. Compare the density of water at the surface and at the bottom of a lake 100 metres deep, given that the compressibility is $\frac{1}{22000}$ per atmosphere of 760 mms. of mercury, and that the specific gravity of mercury is 13.6. (L. 1894.)

41. The barometer changes from 762 to 736.6 cms. between the two weighings of a drying-tube whose mass is 40 gms. The apparent increase in mass due to moisture absorbed is 0.5 gm. Find the error which would be produced if, in the correction for buoyancy, the change in the barometer were neglected; given that the density of air at the first observation is .0012 gm. per c.cm., that the mean density of the drying tube and its contents is 2.4, and that the density of the weights is 8.4. (V. 1893.)

42. One limb of a U-tube consists of a capillary of internal radius 0.05 cm., and the other of a similar capillary of radius 0.2 cm. If this U-tube be partially filled with a liquid of density 1.05 and surface-tension 50 dynes/cm., what will be the difference in the heights of the meniscus in each tube? (V. 1903.)

43. Given that the value of the surface-tension of water is 80 c.g.s. units, calculate its value in millimetre-milligram-minute units. A U-tube with vertical limbs is about half filled with water; calculate the difference in the height of the liquid in the two limbs if one limb is 1 cm. in diameter and the other 1 mm. (V. 1905.)

44. A soap-bubble of 2 cms. diameter is situated within another of 3 cms. diameter. Find the diameter of a single bubble, the interior of which would be at the same pressure as the interior of the smaller bubble. (V. 1902.)

45. If the surface-tension of water is 74 dynes per centimetre, what pressure will it produce in a bubble $\frac{1}{1000}$ cm. in diameter? (L. H. 1902.)

46. A capillary tube is dipped vertically into mercury of surface-tension 540 c.g.s. units, and the surface in the tube (assumed to be part of a sphere) is found to be 1 cm. below the free surface outside. If $g = 981$ c.g.s. units, and the density of mercury is 13.6, calculate the radius of curvature of the meniscus. (V. 1900.)

47. Express a surface-tension of 80 c.g.s. units in ft.-lb.-sec. units, given that 1 lb. = 453.6 gms. (V. 1899.)

48. A capillary 0.1 mm. diameter has its lower end immersed in water, and the water rises in it to a vertical height of 30 cms. Calculate the surface-tension of the water. If the capillary tube were shorter than 30 cms. discuss what would happen. (V. 1896.)

49. A drop of water is placed between two plane pieces of glass which are pushed close together until the film is everywhere x cms. thick and A sq. cms. in area. Find the force with which the plates cling to each other, the surface tension of water being T and its angle of contact with glass zero. (L. 1893.)

50. What will be the pressure in a spherical cavity within a mass of water, if the cavity is .001 cm. in radius and at a depth of 10 cms. below the surface of the water? The pressure of the air on the surface of the water is that due to 760 mms. of mercury, and the surface-tension of water is 78 c.g.s. units. (L. 1895.)

51. Calculate the work done on the film in blowing a soap-bubble from a diameter of 3 cms. to one of 30 cms., if its surface-tension be 45 c.g.s. units. (L. 1897.)

52. Prove that the pressure-excess inside a soap-bubble of radius R is $2gsh \frac{r}{R}$, where h is the height to which the soap

solution of density s could rise in a tube of radius r . Work out the numerical value if the liquid is as dense as water, and if the diameter of the bubble equals the height the liquid could ascend in a tube $\frac{1}{4}$ mm. in bore. (L. 1891.)

53. A vertical glass tube is drawn out at its lower end into a capillary, and a mercury column extends from a point in the capillary to a point 37.5 cms. above it. Find the radius of the mercury meniscus in the capillary, if the surface tension of an air-mercury surface is 540 dynes per cm., and the density of mercury is 13.6. (V. H. 1898.)

54. A bubble of radius 10 cms. filled with air at 15° C. contracts till its radius becomes 5 cms. Find the mass of the air expelled, given that the mass of 1 c.cm. of air at N.T.P. is .0013 gm., and that the surface-tension of soap solution in air is 70 c.g.s. units. (V. H. 1900.)

55. Prove that the saturated vapour pressure outside the surface of a sphere of liquid of radius a exceeds that outside a flat surface of the same liquid by $\frac{\rho}{\sigma - \rho}$ times the difference of pressure inside and outside the spherical surface; σ and ρ being respectively the densities of the liquid and of the vapour surrounding it (The sphere is to be supposed so large that the effect of curvature upon the vapour pressure is small compared with the vapour pressure itself.)

56. If the pressure at a certain level in a vapour obeying Boyle's Law be P , shew that the pressure P' at a level lower than the first by d is given by $P \log_e (P'/P) = g\rho d$, where ρ is the density of the vapour under pressure P . Shew that this equation may also be written $\log_e (P'/P) = gd/R\theta$, where R and θ have the same meanings as in the Gas Laws.

57. From the result of Q. 56 shew that the saturated vapour pressure P' outside a very small sphere of liquid of radius a is given by the equation

$$P \log_e (P'/P) = \frac{\rho}{\sigma} \left(P' - P + \frac{2T}{a} \right),$$

where T represents the surface-tension, P the vapour pressure outside a flat surface, and ρ the corresponding density of the saturated vapour. Shew that $P' - P$ may be neglected in comparison with $\frac{2T}{a}$; also deduce the result of Q. 55 from the foregoing equation.

58. Apply the approximate calculation of Q. 55 to find the value of $P' - P$ for water globules of radius (a) 1 mm., (b) .001 mm., at temperatures 15°C . and 100°C . in each case. It is given that 1 litre of hydrogen at N.T.P. weighs .09 gm., and the molecular weight of water is 18; the saturated vapour may be assumed to obey the gas laws; the density of water is 1 at 15° and 0.7 at 100° ; the surface-tension is 75 dynes per cm. at 15° , and 60 at 100° ; the vapour pressure of water at 15° and 100° is 12.67 mms. and 760 mms. of mercury respectively.

59. Work the previous question by the accurate formula of Q. 57, and thus shew that the result is scarcely affected. ($\log_{10} e = .434$; a pressure of 760 mms. of mercury = 10^6 dynes per sq. cm.)

60. Assuming the surface-tension of water not to vary with the extent of surface, calculate by the methods of Q. 55 and Q. 57 the size of a globule outside which the saturated vapour pressure (a) at 15°C ., (b) at 100° , is double the vapour pressure over a flat surface at the corresponding temperature. Use the data of previous questions.

61. If we have means at our disposal for just detecting variations of pressure equivalent to 0.1 mm. of mercury, calculate by the formula of Q. 55 the radius of the coarsest capillary tube within which the effect of curvature on the saturated vapour pressure of water will be noticeable at 100°C ., assuming the surface-tension of water to be 60 dynes per cm.

62. Calculate the height to which the water would rise in the capillary in the previous question.

63. A bicycle tyre leaks at the valve where it is found that if wetted a bubble one-quarter of an inch in diameter is blown in from 55 to 65 seconds. How long will it take for the pressure to fall to half its amount? The inner diameter of the air-tube may be taken as $1\frac{1}{4}$ inch and the diameter of the axis of the air-tube as $26\frac{1}{2}$ inches. (L. H. 1904.)

SOUND.

64. If an engine whistle appears to change in frequency in the ratio 9 : 8 in passing a hearer, find the ratio of the velocity of the engine to the velocity of sound. (L. 1894.)

65. An engine in a cutting between two bridges is whistling when its velocity is $\frac{1}{20}$ that of sound in air. Compare the frequencies of the echoes from the two bridges to an observer between them. (L. 1896.)

66. A train is approaching a station at the rate of 60 miles per hour, and a whistle giving 1000 vibrations per second is sounded on the train. What will be its apparent frequency to an observer at the station? If a similar whistle is sounded at the station, what will be its apparent frequency to an observer on the train? The velocity of sound is 1100 feet per second. (V. 1901.)

67. A bar of density 8 and length 1 metre vibrating longitudinally gives a fundamental three octaves above middle C (256); find Young's Modulus for it. (L. 1891.)

68. What is the elasticity of a rod a metre long and seven times as heavy as water, which vibrates longitudinally 1200 times a second? (V. 1897.)

69. Calculate Young's Modulus for a given sample of wood from the observations that a yard long of it floats vertically in water with two inches protruding, and emits

when stroked longitudinally a note three octaves above the middle C (256). (L. 1892.)

70. A bar 2 metres long having a section 1 cm. square is supported by its ends and droops 2 cms. for an extra load of 1 kilogram at the centre. Calculate Young's Modulus. If the bar weighs 600 gms., what is the velocity of sound along it? (V. H. 1894.)

71. A rod of specific gravity 0.7 is found to be stretched one millimetre when suspended vertically and loaded with a weight of lead of specific gravity 11, the volume of the weight being equal to that of the rod. What note will the rod alone emit when held in the middle and rubbed longitudinally? (V. 1892.)

72. What is the frequency of the gravest note which can be sounded by a steel wire 2 mms. in diameter and 150 cms. long when stretched by a weight of 50 kilograms? The specific gravity of steel is 7.7. (V. 1901.)

73. A string is under such tension that its length is increased by $\frac{1}{n}$ of its original value. Shew that the frequency of the fundamental longitudinal vibration is \sqrt{n} times the frequency of the fundamental transverse vibration. (L. H. 1895.)

74. One end of a flexible cord, whose mass is 0.1 gm. and length 392 cms., is attached to the end of a prong of a large tuning-fork, and hangs vertically with a weight of 36 gms. at the lower end. When the prong is horizontal and vibrating in a vertical plane the cord breaks up into four segments; calculate the pitch of the fork. (V. 1903.)

75. What is the velocity of sound in a gas, in which two waves of length 1 and 1.01 metres respectively produce 10 beats in 3 seconds? (V. 1902.)

76. Joly's value for the specific heat of air at constant volume is $0.17151 + 0.02788s$, where s is the density. The specific heat at constant pressure is 0.2389 ; find the velocity of sound in air of density 0.001293 , the barometer standing at 760 mms. of mercury of density 13.6 , and the value of g being 981 cms. per sec. per sec. (L. 1901.)

77. A series of closed organ pipes are arranged in order of gradually diminishing length. Any one of the pipes gives seven beats a second with the one next to it. Two beats are heard when the 35th and 1st are sounded together, and five when the 36th and 1st are sounded. What is the length of the longest pipe, if the velocity of sound in air is 340 metres per sec. ? (V. H. 1896.)

78. The velocity of sound in water is 1440 metres per second; calculate the diminution in volume of a litre of water when subjected to an extra pressure of 10^6 dynes per sq. cm. (V. 1900.)

79. Two tuning-forks vibrating 200 times per second are in perfect unison when at the same temperature, but when there is a difference of 10° the Lissajous curves formed by them are found to pass through a complete cycle in 5 seconds, the warmer fork vibrating more slowly. Find the temperature coefficient of the frequency of the fork. (V. 1899.)

80. A tuning-fork gives a particular note at 15° C. It is put in boiling water, and immediately after it is taken out its frequency is lowered one per cent. The coefficient of linear expansion of the metal is 12×10^{-6} . What is the temperature-change in the elastic constant? (L. H. 1902.)

81. Given $\log 2$ and $\log 3$, calculate the intervals between true and tempered fifths, and between true and tempered fourths. (L. H. 1900.)

HEAT.

82. A barometer with a brass scale reads 75.80 cms. at a temperature of 16°C . What will be the reading reduced to 0°C ., if the coefficient of linear expansion of brass is 18×10^{-6} and that of cubical expansion of mercury is 18×10^{-5} ?

(V. 1896.)

83. The metre being defined as the distance between two parallel lines on a platinum bar at 0°C ., and the yard as the distance between two similar lines on a brass bar at 18°C ., express the metre in terms of the yard, given the result of a direct comparison of the two at 18°C . as 1 metre = 1.09393 yds.; coefficient of expansion of platinum = 86×10^{-7} . (V. 1898.)

84. If a crystal have a coefficient of expansion 13×10^{-7} in one direction and of 231×10^{-7} in every direction at right angles to the first, calculate its coefficient of cubical expansion.

(L. 1890.)

85. A crystal having principal coefficients of expansion .000010, .000012, and .000015, weighs 50 gms. in vacuo and 30 gms. in water at 4°C . Find its weight in water at 20°C . of density 0.99827. (V. H. 1898.)

86. Shew that in the case of a water-in-glass dilatometer the temperature of apparent maximum density will be above the true temperature, and find how much mercury should be placed in the bulb of such a dilatometer in order that the

temperature of apparent maximum density may be identical with the true temperature, given that the coefficients of dilatation of mercury and glass near 0°C. are $\cdot 000180$ and $\cdot 000023$ respectively. (V. H. 1902.)

87. Explain how the force that can be exerted by a bar when cooling is connected with its coefficient of linear expansion for temperature and its coefficient of elongation for traction. (V. 1894.)

88. A lump of quartz which has been fused is suspended from a "quartz fibre" and allowed to oscillate under the influence of the torsion of the fibre. If the coefficient of expansion of the material is $0\cdot 0000007$ and the temperature-coefficient of its rigidity is $+0\cdot 00013$, how many seconds a day, or what fraction of a second a day, would a change of temperature of 1°C. make? (L. 1903.)

89. A watch has a non-compensated brass balance-wheel and a steel balance-spring. How many seconds will it lose per day for 1° rise of temperature, if the coefficient of linear expansion of brass is $18 \div 10^6$, and if Young's Modulus for steel at t° is $2 \times 10^{12} \left(1 - \frac{2t}{10^4}\right)$? (L. 1902.)

90. Explain the cause of draught in chimneys. What average excess in temperature in a chimney 136 feet high would under ordinary circumstances produce a pull measured by 1 inch on the water gauge? (L. H. 1902.)

91. Describe Bunsen's calorimeter. Given that 1 c.cm. of ice at 0°C. yields $\cdot 9178$ c.cm. of water at 0°C. , determine the specific heat of a body whose mass is 1 gm. and temperature 100°C. which when dropped into the instrument produces a shrinkage of 6 cubic millimetres. (Latent heat of fusion = 80.) (V. 1905.)

92. Determine the ratio of the specific heats of air from the following data. Velocity of sound = 34215 cms. per sec. in air at 750 mms. and 17°C. ; density of air = $\cdot 00129$ gm. per c.cm. at N.T.P.; coefficient of expansion of air = $\frac{1}{273}$; $g = 981$ cms. per sec. per sec.; density of mercury = $13\cdot 6$ gms. per c.cm. (V. 1898.)

93. Calculate the specific heat of a gas from the following data. The gas passed through the calorimeter was supplied by a vessel of capacity 30 litres. At the beginning of the experiment the pressure in the vessel was 7 atmospheres, while at the end it had fallen to 3 atmos. The temperature of the gas was such that under atmospheric pressure its density would have been $\cdot 0012$. The difference in the temperature of the gas on entering and leaving the calorimeter was 208° , and the corrected rise of the temperature of the calorimeter was $10\cdot 2$. The water equivalent of the calorimeter and contents was 700 gms. (V. 1893.)

94. The thermal conductivity of a substance is represented by 1 in c.g.s. units; what will be its numerical value in millimetre-milligram-minute units? (V. 1896.)

95. A tin cylinder 40 cms. in diameter and 50 cms. long is covered all over by a layer of felt 0.33 cm. thick. Steam at 100°C. is passed through the cylinder, the external temperature being 20°C. , and water is found to accumulate at the rate of 3 grams per min. If the Latent Heat of steam at 100° is 537, find the conductivity of the felt. (L. 1897.)

96. A small quantity of heat is to be measured (i) by Bunsen's ice calorimeter, (ii) by Favre and Silbermann's method, in which the heat is imparted to the mercury in a large thermometer bulb, (iii) by the same method with ether in place of mercury. Compare the sensitiveness of the three ways. Latent heat of fusion of ice = 80; density of ice = $0\cdot 92$;

specific heat of mercury = $\frac{1}{30}$, of ether = $\frac{1}{2}$; coefficient of expansion of mercury = $\cdot 00018$, of ether = $\cdot 00165$; density of mercury = $13\cdot 6$, of ether = $0\cdot 7$.

97. Assuming Newton's Law of Cooling, compare the rates of loss of heat in vacuo of two copper spheres whose radii are 10 cms. and 5 cms., their temperatures being respectively 20° and 10° above that of the enclosure in which they are placed. Compare also the rates of fall of temperature. (Both spheres conduct heat perfectly.)

98. Give the relative rates of cooling of two similar bodies at 327° and 127° C. respectively, the surrounding bodies being all at 27° , (i) on Newton's law of cooling, (ii) on the supposition that radiation is proportional to θ^4 and that convection may be neglected. (V. 1899.)

99. Calculate from the following data what would have been the final temperature of a calorimeter and its contents after the introduction of a hot body, had all loss by radiation been avoided. Initial temp. of air and water = $15^\circ 3$ C.; after introducing the hot body, the temp. at the end of successive minutes was $18\cdot 8$, $20\cdot 0$, $20\cdot 6$, $20\cdot 7$, $20\cdot 7$, $20\cdot 6$, $20\cdot 5$, $20\cdot 4$, $20\cdot 3$, $20\cdot 2$, $20\cdot 1$, $20\cdot 0$. (V. H. 1893.)

100. Find the weight of a litre of moist air at 20° C., the dew-point being 11° and the barometer standing at 76 cms. (Vapour pressure of water at $11^\circ = 1$ cm. approx.; density of dry air at 76 cms. pressure and 20° C. = $\cdot 00120$; relative density of aqueous vapour and air = $0\cdot 624$.) (V. 1895.)

101. Calculate the dew-point for air $\frac{3}{4}$ saturated at 18° C., given that for pressures of 6, 9, 12, 15 mms. of mercury respectively, the corresponding boiling-points of water are 4° , 10° , 14° , and 18° C. (V. 1899.)

102. A mixture of air and of the vapour of a liquid in contact with excess of the liquid is contained in a vessel of

constant volume. At a temperature of 15°C . the pressure in the vessel is 70 cms. of mercury, at 30° , 45° , and 60° the pressures are 88, 110, and 145 cms. respectively. If at 15°C . the vapour pressure of the liquid is $15\cdot4$ cms., what is it at 30° , 45° , and 60° ? (L. 1897.)

103. Assuming Boyle's Law to hold, find the weight of dry air in a vessel containing 300 c.cms. of air saturated with aqueous vapour at 20°C . and subject to a total pressure of $73\cdot74$ cms. of mercury; the density of air at N.T.P. being $\cdot001293$ gm. per c.cm. and the maximum pressure of aqueous vapour at 20° being $1\cdot74$ cm. of mercury. (V. 1897.)

104. Calculate the average molecular velocity in the case of a gas whose density at a pressure of 50 cms. of mercury is $0\cdot001$ gm. per c.cm. (V. 1896.)

105. Calculate the molecular velocity (of mean square) in the case of a gas whose density is $1\cdot4$ gm. per litre at a pressure of 76 cms. of mercury. Density of mercury = $13\cdot6$; $g = 981$ cms. per sec. per sec. (V. 1905.)

106. Shew how the mean free molecular path in a gas may be obtained when the viscosity is known, and find the mean free path in air regarded as a uniform gas, given that the pressure is 10^6 dynes per sq. cm., the density 12×10^{-4} gm. per c.cm. and the viscosity 17×10^{-5} c.g.s. unit. (L. H. 1901.)

107. If a gas is suddenly compressed to $\frac{1}{10}$ th of its original volume, obtain an expression from which the resulting temperature could be calculated. (V. 1905.)

108. (i) Air at 15°C . is suddenly compressed to $\frac{1}{10}$ th of its volume; find the temperature to which it will rise. (ii) If the compression raises the pressure to 10 times its original value, find the final temperature. (L. 1890.)

109. What pressure would be required to compress dry air at the standard pressure and temperature into $\frac{1}{50}$ of its volume (*a*) slowly, (*b*) suddenly; and what rise of temperature would result? (Ratio of specific heats, 1.4.) (V. 1892.)

110. If ordinary dry air at ten atmospheres and 15°C. is suddenly released, find with the help of logarithms the new temperature. (V. 1897.)

111. It was found by Noble and Abel that the explosion of 1 gm. of gunpowder in a closed space of 1 c.cm. exerted a pressure of 6300 atmospheres and left a liquid residue of 0.6 c.cm.; while the gaseous products drawn off to 0°C. and 760 mms. occupied 280 c.cms. Deduce from these data the temperature attained in the explosion. (L. H. 1901.)

112. Shew how to calculate the difference between the two specific heats of a perfect gas, given that a gram of it occupies 1000 c.cms. at a temperature of 27°C. and one megadyne per sq. cm. pressure. ($J = 4.2 \times 10^7$ ergs per calorie.) (V. 1895.)

113. Calculate the intrinsic energy of air at 15°C. and 750 mms. pressure, assuming the ordinary values of such physical constants as you require.

114. From what height must a lump of ice at 0°C. fall in order to melt itself, supposing it possible to concentrate all the energy of the fall in the lump? (L. 1891.)

115. Find the velocity of a lead bullet which warms itself 250°C. by striking an unyielding target, the whole heat generated by the impact heating the lead. (Specific heat of lead = .033.) (V. 1899.)

116. Two bars of iron, each weighing 50 kilograms, are supported horizontally, each by two vertical cords 100 cms.

long, all four strings being in the same vertical plane, in such a way that the axes of the bars are in the same horizontal line, and their nearer ends a short distance apart. In the space between the bars and just touching each is placed 1 kilogram of lead of specific heat $\frac{1}{30}$. One of the bars is pulled out in the plane of the cords till they are inclined at 60° to the vertical, and then the bar is released so as to impinge directly on the lead and crush it. After the impact the horizontal amplitude of oscillation of the whole system is found to be 20 cms. on each side of the position of equilibrium. Calculate the rise in temperature of the lead, supposing all the heat produced by the impact to be confined to it.

(V. H. 1901.)

117. Calculate the internal and external work done in vaporising a gram of water at 100°C ., given that the latent heat of evaporation is 536; the value of Joule's equivalent is 42×10^6 ergs per calorie, the volume of a gram of steam at 100°C . and 760 mms. pressure may be taken as calculable by the gas laws from its molecular weight, the specific gravity of mercury is 13.6, and the value of g is 981 cms. per sec. per sec.

118. The latent heat of steam at 100°C . is 536; calculate what fraction of this heat is used up in performing external work during vaporisation, assuming the density of steam at 100°C . to be .0007, and an atmosphere to be 10^6 dynes per sq. cm.

(V. 1900.)

119. Assuming that steam obeys the gas laws, shew that the work done in changing the volume of 1 gm. of steam at 100°C . and 760 mms. to the volume at 101°C . and 787 mms. (the saturation pressure at 101°C .) is more than enough to supply the heat needed for the rise in temperature; the specific volume of steam at 100° being 1700, and its specific heat at constant pressure 0.48.

(L. H. 1894.)

120. Calculate the change of entropy when a gram of water at 30°C. is mixed with 2 gms. at 0° , assuming that the specific heat of water is unity at all temperatures.

121. Calculate the change of entropy when a gram of steam at 100°C. cools to water at 0° , assuming that the latent heat of vaporisation is 536 and that the specific heat of water is 1 at all temperatures.

122. An engine of eight H.P., supposed reversible, works between the temperatures 100°C. and 15°C. How much water should be evaporated in the boiler per hour, given the appropriate values of Joule's equivalent and the latent heat of steam?

123. Coal is burnt at a temperature of 1527°C. , and the heat is used (a) to warm the air of a room at 7°C. , (b) to drive a reversible heat engine whose condenser is at the temperature of the room. In case (b) the heat engine drives backwards another reversible heat engine working between temperatures 7°C. and 27°C. , and the heat escaping from the latter is used to warm the air of the room. Compare the two processes (a) and (b) from the point of view of economy.

124. Calculate the volume of a gram of steam at 100°C. , given that the latent heat of evaporation of water at $100^{\circ}\text{C.} = 535$, and the change of boiling point is $0^{\circ}\cdot 37^{\circ}\text{C.}$ per cm. of mercury pressure. ($J = 4\cdot 2 \times 10^7$.) (V. H. 1900.)

125. Calculate the change in the boiling point of water for a fall of the barometer from 76 to 75 cms., assuming that a gram of steam occupies 1700 c.cms., and taking the usual values for other constants. (V. H. 1894.)

126. If sulphur has a density 2.05 just before, and 1.95 just after melting, the melting point being 115°C. and the latent heat 9.3, find the alteration in melting point per atmosphere change of pressure. (L. H. 1893.)

127. A spring when pulled out at 15°C . is found to require to be supplied with heat equivalent to $\frac{1}{10}$ of the work done in its extension to prevent its temperature from falling. Find the temperature-coefficient of pull for a given extension.
(L. H. 1896.)

128. The surface tension of a water-air surface is 77 dynes per cm. at 0°C . and 60 dynes per cm. at 100°C . Compare the amount of work required to blow a bubble of given size isothermally and adiabatically.
(V. H. 1901.)

LIGHT.

129. The mean interval between successive eclipses of Jupiter's second satellite is 42 h. 28 m. 42 s. Calculate the greatest and least intervals, given that the velocity of light is 3×10^{10} cms. per sec., and that the earth's mean orbital velocity = 3×10^6 cms. per sec. (V. H. 1900.)

130. Assuming the satellite of a planet to revolve once in 48 hours, calculate the greatest and least interval between two successive eclipses of the planet as observed from the earth. Distance of sun from earth = 150×10^6 kilometres; velocity of light = 3×10^5 kilometres per sec. (V. H. 1898.)

131. The wave-length of a line in the solar spectrum is found to be 6×10^{-5} cm. at the sun's pole, and is changed by 4×10^{-10} cm. at that part of the sun's equator which is moving in the line of sight. Calculate the linear velocity at the equator, if the velocity of light is 3×10^{10} cms. per sec. (V. 1899.)

132. In the spectrum of a star the more refrangible component of the double sodium line is found to coincide in position with the less refrangible component of the pair in the solar spectrum. Calculate the velocity of the star, and state whether it is moving towards the earth or away from it. (The wave-lengths of the two sodium lines are 5889 and 5895 "tenth metres" respectively.) (V. H. 1898.)

133. A concave spherical reflector is placed with its axis vertical so as to form a shallow tray. The position of the centre of curvature is 100 cms. above the surface of the tray. Calculate the position of the apparent centre of curvature, *i.e.* that point on the axis for which an object and its image are coincident, when the tray is filled with water of index $\frac{4}{3}$.

(V. 1900.)

134. You are given an achromatic objective of 12 inches focal length, and are asked to construct an inverting telescope focussed for infinity and magnifying twelve times. State what lenses you would require for the eye-piece. (V. 1903.)

135. Two telescopes placed one behind the other magnify 10 and 20 times respectively; each has been so adjusted that the rays from an object at infinity emerge parallel. Prove that the optical system so obtained will be a telescope, and find its magnifying power (α) when the eye-pieces are together, (b) when the objectives are together, (c) when an eye-piece and an objective are together.

(V. 1899.)

136. A telescope with aperture 4 inches is provided with two eye-pieces magnifying 10 times and 100 times respectively. It is used to view (i) a star, (ii) a luminous surface, such as the sky by day. Supposing the eye to have an effective aperture of $\frac{1}{8}$ inch, what will be the ratio of the brightnesses (i) of the star, (ii) of the sky, when seen by the naked eye and with the telescope, with each of the eye-pieces? Which eye-piece would be best for seeing a star by day?

(L. H. 1902.)

137. If an achromatic converging lens has to be constructed of two materials, one of which has refractive indices 1.36 and 1.38 for red and blue rays, while the corresponding indices of the other are 1.78 and 1.80, which material should be used for the concave lens of the combination? If the

compound lens is to have a focal length of 50 cms., what should be the focal lengths of the two constituents?

(V. 1899.)

138. Calculate the focal lengths of the separate lenses in an achromatic combination for which the focal length is to be 30 inches from the following spectroscopic measurements:—crown glass, $\mu_c = 1.516$, $\mu_F = 1.525$; flint glass, $\mu_c = 1.619$, $\mu_F = 1.636$.

(V. 1891.)

139. Find the power of the single lens equivalent to two thin coaxial lenses of powers p_1 and p_2 separated by an interval d .

(L. H. 1899.)

140. A person whose range of distinct vision is between 10 and 20 cms. respectively puts on spectacles which remove the near point to 20 cms.; what is the focal length of the spectacle lens, and what is the greatest distance to which he will see distinctly with the help of the spectacles?

(V. 1895.)

141. A double convex lens whose surfaces are equally curved is 3 inches in diameter and 0.1 inch thicker at the centre than at the circumference. Its refractive index is 1.6; find its focal length.

(V. 1889.)

142. A converging lens has one face flat and the radius of the other is 2 ins.; the thickness of the lens is 1.2 ins. It is found that an object 8 ins. away from the convex face forms an image of the same size at 7.2 ins. from the flat face. Calculate the refractive index of the glass.

143. The lens of Q. 142 is used to form an image of a small object on the axis at 11.2 ins. from the flat face; calculate the distance of the image from the curved face.

144. Parallel light is incident on the convex side of a plano-convex lens 1 in. thick and 3 ins. in diameter, its edges being sharp and its refractive index equal to 1.5; find how far from the flat face the light will be brought to a focus.

145. Calculate the position of the principal points of a double convex lens whose radii of curvature are 20 and 30 cms. respectively, the glass being 1 cm. thick and having a refractive index of 1.5. (V. H. 1893.)

146. Three convex lenses are each 1 in. thick and 3 ins. in diameter with sharp edges, and are made of glass of refractive index 1.5. The first has one flat face, the second has faces of equal curvature, the third has a concave face which exactly fits the faces of the second. Calculate for each lens the focal length and the distance of the principal foci from the corresponding faces.

147. Determine the focal length, radii of curvature, and refractive index of a biconvex lens 1 inch thick from the following axial observations. An object 6 ins. from the nearer face *A* gives a real image 8 ins. from the other face *B*; a bright point at $1\frac{1}{7}$ in. from *B* and $2\frac{1}{7}$ ins. from *A* gives an image by reflection coinciding with itself; the same thing happens if the bright point is $1\frac{1}{2}$ in. from *A* and $2\frac{1}{2}$ ins. from *B*.

148. The lens of Q. 144 has air on one side and water (refractive index $\frac{4}{3}$) on the other. Determine the position of the nodal points according as the water adjoins (i) the flat or (ii) the curved face; also of the focal points.

149. An object is situated on the axis of the lens of Q. 148 at a distance of 6 ins. from the nearer face. Determine the distance of the image from the nearer face in each of the possible cases; also find the magnification.

150. If in a normal eye the distance of the "second nodal point" from the retina is 15 mms., what will be the area covered on the retina by the image of a circular disc 30 cms. in diameter, the centre of which is 2 metres distant from the eye in the direct axis of vision and whose plane is at right angles to this axis? (L. 1898.)

151. An equiconvex glass lens (refractive index $\frac{3}{2}$) with sharp edges is 1 in. thick and 3 ins. in diameter. It separates air on one side from water (refractive index $\frac{4}{3}$) on the other. Determine the position of the nodal points and focal points.

152. An object is placed on the axis 6 ins. from the nearer face of the lens of Q. 151, (i) on the side where there is air, (ii) in the water. Find the position of the image in each case, and also the magnification.

153. A zone-plate exposed to parallel rays of light incident normally upon it brings the light to a focus 5 inches away; if the wave-length of the light is $\cdot 00002$ inch, what is the outer radius of the first opaque zone, the centre being transparent? If the plate receives light from a point on its axis 15 ins. away, where will be the conjugate focus?

154. A zone-plate intended for use with the lantern has the outer radius of the first opaque zone equal to 2 mms.; how far off will be its principal focus for light of wave-length $\cdot 00005$ cm.?

155. The object-glass of a telescope is covered by a screen containing two narrow parallel apertures 12 cms. apart, and an illuminated slit placed parallel to the apertures is looked at through the telescope. If the light from the slit is of wave-length 5×10^{-5} cm. and the telescope magnifies 20 times, calculate the angular distance between the two central dark bands. (V. 1899.)

156. Describe Fresnel's biprism experiment. The wave-length of sodium light is 5892×10^{-8} cm.; find the distance between the interference bands supposing the distances between slit and biprism, and between biprism and screen ~~to be each~~ 50 cms. The angle of the biprism is 179° ~~and its index of refraction is 1.5.~~

157. Describe in detail how you would arrange an optical bench so as to exhibit interference fringes with a biprism. How would you measure the distance between the two virtual images of the slit formed by the biprism? If this distance is 0.2 cm., the wave-length of the light is 5900 tenth-metres, and the screen is 100 cms. from the slit, calculate the distance between two consecutive fringes. (V. 1903.)

158. What relation exists between the diameters of successive Newton's rings when a convex lens in optical contact with a flat surface is viewed by monochromatic light? If two such successive rings have diameters of 3 mms. and 3.1 mms. when viewed normally, calculate the radius of curvature of the convex surface. (V. 1903.)

159. The convex face of a lens rests on a flat plate, and the Newton's rings formed are viewed normally by reflected light from a sodium flame. The wave-length is 5897 tenth-metres, and the diameters of two consecutive rings are 1 cm. and 1.01 cm.; calculate the radius of curvature of the lens.

160. Calculate approximately (neglecting multiple reflections) the thickness of an air film which obliterates light of wave-length 5×10^{-5} cm. from the reflected beam, the rays in the film passing at an angle of 60° to the normal. (V. H. 1896.)

161. Calculate the thickness of an air film between plates of a substance having a refractive index of $\sqrt{2}$, if a wave-length of 5×10^{-5} cm. as measured in air is absent in the reflected beam, the angle at which the ray strikes the film being 30° . (V. 1902.)

162. Calculate the thickness of a soap-film (refractive index $\frac{4}{3}$), which, when white light is incident on it at 45° , gives a dark band in the reflected light whose centre is found by the spectroscope to correspond to wave-length 6×10^{-5} cm.

163. The thickness of a certain object is known to be between 1.015 and 1.020 mm. When it is expressed in terms of four standard wave-lengths (6438.9, 5086.3, 4800.0, and 4678.9 "tenth-metres") the fractions of a wave are found by experiment to be .61, .14, .55, .51 respectively. Shew that (within the possible limits) these fractions agree best with the integers 1581, 2002, 2121, 2176 respectively; and hence, calculate the thickness accurately. (V. H. 1905.)

164. A beam of plane polarised light is changed into circularly polarised light by passing through a slice of crystal 0.003 cm. thick; calculate the difference in the refractive index of the two rays in the crystal, assuming this to be the minimum thickness that will produce the effect, and that the wave-length is 58×10^{-6} cm. in air. (L. 1897.)

MAGNETISM.

165. The value of H at Kew is about 0.18 c.g.s. unit; express this in mm.-mgm.-min. units.

166. At what points do the lines of force due to an ideal magnet run at right angles to its magnetic axis?

167. A magnet of moment M is placed (i) East and West, (ii) North and South; find the points in the same horizontal plane as the magnet at which the earth's horizontal field is just annulled.

168. A solid cylindrical bar of magnetised steel, 10 cms. long and 1 cm. in diameter, vibrates once in 10 secs. due to the earth's horizontal field when suspended so as to oscillate about an axis perpendicular to its length. If the density of steel is 8, and $H=0.18$, calculate the value of the magnetic moment. (V. 1903.)

169. A weight of 0.1 gram, acting at 7 cms. from the centre of gravity of a dip magnet, causes the magnet to set horizontally. The same weighted magnet oscillating in a horizontal plane has a time of oscillation of 6.28 seconds. Calculate to one per cent. the sine of the angle of dip, if the moment of inertia of the combined system is 700 c.g.s. units. (V. 1902.)

170. A small magnet placed end-on at a metre from a declination magnetometer deflects it through $1^{\circ} 30'$

($\tan 1^\circ 30' = \cdot 0262$); and when placed end-on at 2 metres from a bifilar magnetometer deflects it through 7 divisions of its scale. Calculate the percentage of change in the horizontal component of the earth's magnetic intensity that is represented by each division of the scale of the bifilar, assuming the average value to be $H = \cdot 1811$. (L. 1897.)

171. Find an expression for the couple which a small magnet B exerts on a second small magnet A , whose axis produced bisects that of B at right angles. Find also the couple exerted by A upon B , and reconcile the result with the axiom that "action and reaction are equal and opposite." (V. 1905.)

172. Two magnets of the same steel, dimensions $10 \times 3 \times \cdot 5$ and $20 \times 4 \times \cdot 7$ respectively, were found, when pivoted horizontally in the earth's field so as to oscillate about their shortest axes, to swing at the same rate. Compare the intensities of magnetisation of the two. (V. 1898.)

173. Two short magnets, each of moment M , are so placed as to make angles of 45° with the line joining their centres. Find the strength and direction of the magnetic field at the point midway between them in each of the possible cases.

174. If two short magnets of equal moment are placed with the line joining their centres along the axis of one and perpendicular to the axis of the other, calculate the intensity and direction of the field at the middle point of the joining line. (V. 1900.)

175. Two equal linear magnets are placed so that their directions are at right angles to each other. Find the direction and intensity of field at the point towards which both magnets are directed. (Avoid choosing a special case.) (V. 1899.)

176. Three short magnets, each of moment M , are placed at the corners of an equilateral triangle whose side is a , two lying along a side of the triangle with the North poles towards each other, while the third is perpendicular to that side. Find the strength and direction of the magnetic field at the centre of the triangle in each of the possible cases.

177. Two short magnets, each of moment M , occupy two corners of an equilateral triangle, and lie along the side joining them, its length being a . Find the strength and direction of the magnetic field in each possible case, (i) at the third corner, (ii) at the centre of the triangle.

178. The magnets in Q. 177 are turned so that each lies along the side joining it to the third corner. Find the strength and direction of the magnetic field there and at the centre of the triangle in each possible case.

ELECTRICITY.

179. Two small balls each weighing 0.1 gm. are suspended by silk fibres 100 cms. long from the same point. A charge of electricity is now communicated to them so that they separate till the distance between their centres is 4 cms. Calculate the charge on each, assuming the electricity to be uniformly distributed over the surfaces. (V. 1896.)

180. Calculate the capacity of a pair of large parallel conducting plates of area A when the distance between them is small and represented by d . Shew further that the field between the plates remote from the edges is uniform, and find its value for unit potential difference. (V. 1905.)

181. A thunder-cloud is over still water. What is the electrostatic surface-density of the charge on the water if under the cloud it rises 0.1 cm. above its previous level? (L. 1893.)

182. If the area of each of the plates of a parallel plate condenser is 1000 sq. cms., and if their potential difference is 100 volts with an air-space of 0.1 cm. between them, calculate in ergs the energy stored in the condenser. (Velocity of light = 3×10^{10} cms. per sec.) (V. 1900.)

183. Calculate to one per cent. the force in gms.' weight exerted by one of the two plates of a condenser on the other,

the plates being at a distance of 0.01 cm., the area of each plate being 100 sq. cms., and the difference of potential being 1000 volts. The effect of the edges may be neglected.

(V. 1895.)

184. What potential would enable the tension on an insulated sphere of 3 cms. radius to balance $\frac{1}{2000}$ of an atmosphere?

(L. 1891.)

185. A sphere 24 cms. in diameter has a charge of 15 units. When connected to a quadrant electrometer it gives a deflection of 48 divisions. It is then joined by a fine wire to a sphere of 12 cms. diameter, and the deflection is now only 38. Find the capacity of the electrometer and the charge on the small sphere.

(L. 1897.)

186. A charged particle is placed near a conducting spherical surface. Shew that even though the sphere and the particle are charged with electricity of the same sign, the particle will be attracted to the sphere unless its distance from the surface of the sphere exceeds a certain value. Find an expression for this value.

(L. H. 1895.)

187. A microfarad condenser has its plates charged and is then allowed to leak through a very high resistance R . If the potential difference falls to half its original value in 1 minute, find the value of R , given that $\log_e 2 = 0.69$.

(V. H. 1896.)

188. What is the value (a) in electromagnetic c.g.s. units, (b) in coulombs, (c) in electromagnetic mm.-mgm.-min. units, of a quantity of electricity whose electrostatic measure is 250 c.g.s. units?

(L. H. 1899.)

189. Calculate in terms of a microfarad the capacity of an air condenser in which the plates are parallel and each a sq. metre in area, the distance between them being 1 millimetre.

(V. 1891.)

190. A condenser of capacity 1 microfarad is charged by means of a Daniell cell of E.M.F. $1\frac{1}{2}$ volt. If the charge given to it were used to raise metal spheres of 5 cms. radius to a potential of 20,000 volts, how many such spheres could be charged?

191. An electric motor develops 5 horse-power in its shaft with a current of 45 amperes at 100 volts. What is the efficiency of the motor, a horse-power being taken as equivalent to 746 watts? (V. 1889.)

192. Electrical energy is sold at the rate of 4d. per kilowatt-hour. The mechanical equivalent of the heat given by the burning of coal worth 4d. is 10^8 foot-lbs. Compare the prices of the two forms of energy. Why is electrical energy so much dearer than coal energy? (1 H.P. = 746 watts.) (L. 1902.)

193. How many ergs correspond to 1000 watt-hours? A lamp of 400 candle-power requires $2\frac{1}{2}$ watts per candle; how many amperes will it take at 100 volts? What will be the loss of E.M.F. on passing the current through a conductor whose resistance is 0.2 ohm? (V. 1890.)

194. (i) Calculate the value of the energy stored in a condenser formed of two parallel plates 1 metre square separated by 1 mm. of air, the potential difference being 100 volts; the cost of a Board of Trade unit (1000 watt-hours) is 4d. (ii) A condenser built up in this way is to store a pennyworth of energy; calculate the combined thickness of all the air-gaps.

195. Twenty-one calories are produced in 5 minutes in a wire by a current of one-sixth of an ampere. Calculate the number of watts required to keep up this current and the difference of potential at the ends of the wire. (1 calorie = 4.2 joules.) (V. 1899.)

196. Calculate the power required to light 80 incandescent lamps, if the E.M.F. required be 65 volts and the current required by each be 0.8 ampere. If the lamps be all in parallel and the leads have a resistance of 0.5 ohm, calculate the power wasted in them. (L. 1892.)

197. The resistance of a copper wire and a column of electrolyte in series is 100 ohms at 0°C . and 90 ohms at 20°C . If the temperature coefficient of resistivity of copper is .004, and that of conductivity of the electrolyte is .06, find the resistance of the copper wire. (V. H. 1900.)

198. A current is passed between two plates immersed in a solution of copper sulphate and measured by a tangent galvanometer. The deflexion of the galvanometer is 45° , and after an hour it is found that 216 mgms. of copper have been deposited. Being given that unit current deposits 19.6 mgms. per minute, find what current would cause a deflexion of 60° . (V. 1892.)

199. If the chemical action going on in a battery results in the formation of 500 calories of heat for every gram of metal dissolved, and if the quantity of electricity set in motion by the solution of a centigram of metal is 0.5 c.g.s. unit, what is the E.M.F. of the battery (assumed to be calculable from the above data)? (V. 1897.)

200. The electro-chemical equivalent of zinc is 0.00034 gram per ampere-second. Find the cost of the zinc used in a primary battery for each H.P. per hr., if zinc costs p pence per kilogram, and if the cell in which the zinc is used gives V volts. (1 H.P. = 746 watts.) (L. 1895.)

201. A normal Daniell's cell has an E.M.F. of 1.07 volt and resistance 2 ohms. Its terminals are connected by two wires in parallel of 3 and 4 ohms. Assuming that the electro-chemical equivalent of copper is .000328 gram per coulomb,

calculate the weight deposited in the cell, and also the heat developed (*a*) in the cell, (*b*) in each of the wires, during one hour of working of the cell. (V. 1897.)

202. The heat of combustion of hydrogen and oxygen to water is 34200 water gram units for each gram of hydrogen burnt. A c.g.s. unit current decomposes in one second 0.000945 gm. of water. The mechanical equivalent of heat being 4.2×10^7 , find in volts the smallest E.M.F. which can decompose water. (V. 1889.)

203. If a mirror galvanometer can just detect 10^{-11} ampere, and if the smallest visible bubble of mixed gases in a water voltameter is a cubic millimetre, how long will it be before a current which just perceptibly deflects the galvanometer will produce perceptible electrolysis?

204. To what potential would a sphere of radius 10 cms. need to be charged, if by discharging it through a voltameter it is desired to liberate a bubble of hydrogen just perceptible with the aid of a lens, say a millionth of a milligram? (1 coulomb liberates 0.00001035 gm.)

205. A current of electricity is passed through a wire in a calorimeter, a tangent galvanometer, and a voltameter. When the tangent galvanometer shews a deflection whose tangent is .7431 the calorimeter gains 5 calories per second and .00641 gm. of copper is deposited per minute in the voltameter. When the current is increased till .01282 gm. of copper is deposited per minute, calculate the tangent of the deflection of the galvanometer and the number of calories per second developed in the calorimeter. (V. 1894.)

206. A battery of 5 cells, each of which has an E.M.F. of 2 volts and an internal resistance of .03 ohm, is connected (*a*) in series and (*b*) in parallel, the current passing through a wire of resistance 0.1 ohm. Calculate the heat developed in the wire in each case. (V. 1895.)

207. Two single-needle galvanometers, A and B , are made geometrically similar in all respects, the linear dimensions of A being n times those of B . The magnetic fields are so adjusted by exterior magnets that the periods of vibration of the needles are equal. What is the ratio of the angles of deflection when the same steady current is sent through each? (L. H. 1902.)

208. Find an expression for the magnetic potential of a circular coil of n turns of mean radius r , carrying a current of C amperes, at a point situated on the axis at a distance x from the centre of the coil. Deduce from this an expression for the force exerted by the coil on a magnet pole of m units placed at the point in question. At what point on the axis is the rate of change of this force a maximum for a given small change of x ? A coil of 40 turns like that of a tangent galvanometer of mean radius 12 cms. carries a current of 10 amperes. What is the intensity of the field in c.g.s. units which it produces at a point 50 cms. along its axis from the centre of the coil? (L. H. 1897.)

209. A small galvanometer needle, swinging freely under the earth's force alone, makes three oscillations per second. The control magnet of the galvanometer is replaced and adjusted until the needle makes one oscillation in 3 seconds. A millimetre scale is fitted at a distance of a metre from the mirror; find the intensity of the magnetic field at the centre of the galvanometer due to unit current, if a current of 10^{-4} ampere produce a deflection of 20 divisions, the value of H being 0.172. (V. 1893.)

210. A current traverses a tangent galvanometer of 30 turns of mean radius 8 cms. and produces a deflection of 30° at a place where $H = 0.17$. It then traverses a coil of wire of resistance 5 ohms placed in a calorimeter. Calculate the rate

of production of heat assuming $J = 42 \times 10^6$. If the deflection increased to 45° , how would the rate of heating be affected? (V. 1896.)

211. A solenoid of 1000 turns is wrapped uniformly in a single layer on a tube 40 cms. long and 6 cms. in diameter. If the wire carries a current of 0.1 amp., calculate (to three significant figures) the intensity of the field at the middle point of the axis. (V. H. 1901.)

212. Two equal circular coils of wire, each of 10 turns, are mounted parallel to one another and co-axially, and at a distance apart equal to the radius of either. Calculate the intensity of field at the point on the axis midway between the coils, when one ampere flows in the same sense round each coil. (V. 1896.)

213. A square coil of wire of 20 turns and 10 cms. in the side is suspended vertically in the magnetic meridian. What current in amperes must flow through it if the initial torque due to the earth's field be 100 c.g.s. units, the earth's horizontal magnetic field being 0.18? (V. 1898.)

214. A vertical ring of radius a carries a current i and is in equilibrium in the earth's field. Find the work necessary to twist it round a vertical axis into the meridian. (L 1896.)

215. A circular coil of 300 turns of mean radius 10 cms. rotates 10 times per second about a diameter, in the centre of a long solenoid having 5 turns of wire per cm. traversed by a current. The axis of rotation of the coil is perpendicular to the axis of the solenoid. At the instant when the E.M.F. in the coil is a maximum, its ends are connected through a galvanometer to two points A and B on a wire in series with the solenoid, and there is no deflection of the galvanometer. Find in absolute units the resistance between the points A and B . (V. H. 1897.)

216. A coil rotates about a vertical axis 25 times a second, its radius being 16 cms., and the number of turns in it 313. A deflection of 39.7 cms. is observed on a scale at a distance of 252 cms. from the deflected magnet which is suspended at the centre of the coil. Neglecting all corrections, calculate the resistance of the coil. (V. H. 1892.)

217. The sides AB , BC , CD , DA of a square have resistances 1, 3, 7, and 5 ohms respectively; B and D are also joined by a resistance of 2 ohms. Find the equivalent resistance between A and C .

218. A framework is made out of six pieces of the same wire forming a square and its two diagonals. It is fixed together at the angles of the square and at the crossing-point of the diagonals. What is the equivalent resistance of the frame between the two ends of the same diagonal? (V. 1902.)

219. In a Wheatstone net the four arms are 1, 2, 3, 4 ohms respectively, and the galvanometer resistance is 5 ohms. If the battery maintains a p.d. of 1 volt between the corners to which it is joined, find the current in the galvanometer in the two possible cases.

220. In a Wheatstone net the four resistances are equal and the galvanometer is replaced by a battery of the same E.M.F. as the one already present. If the resistances in the battery circuits are each the same as those in the other four arms, find the currents in the various branches. (V. 1899.)

221. A network is arranged as in Mance's experiment. The resistance of the battery, of the galvanometer, and of each of the other three arms, is 4 ohms, and the E.M.F. of the battery is 2.2 volts; find the current in the galvanometer (i) with the key open, (ii) when it is closed. If the resistance of the arm opposite the battery is altered to 5 ohms, find the new values of the current. (L. 1898.)

222. Two storage cells are connected in parallel to a circuit of 1 ohm resistance. Their E.M.F.'s are 2.05 and 2.15 volts, and their internal resistances .05 and .04 ohm respectively. Find the current through each cell. (V. H. 1900.)

223. A battery of 5 cells in series and a battery of 4 cells also in series have their positive terminals joined together, and to one end of a wire of 10 ohms resistance, the other end being joined to the negative terminals of both batteries. If the E.M.F. of each cell is 1 volt, and the resistance of each is 1 ohm, determine the current in each portion of the circuit.

(V. 1894.)

224. A storage cell (2 volts and $\frac{1}{10}$ ohm), a Daniell cell (1 volt and 2 ohms), and a Bichromate cell (1.8 volt and $\frac{1}{2}$ ohm) have their positive terminals all joined, and also their negative terminals. Calculate the current through each cell.

(V. 1905.)

225. Two cells, one a Daniell of E.M.F. 1 volt and resistance 2 ohms, the other a Leclanché of E.M.F. 1.3 volt and resistance 3 ohms, have their positive terminals both joined to one end of a wire of resistance 5 ohms, while their negative terminals are both joined to the other end. Calculate the current in the wire and in each cell.

(V. 1900.)

226. If the vertical magnetic force of the earth be .4342, calculate the E.M.F. in volts generated in a straight bar 2 metres long carried by a railway train at a rate of 80 kilometres per hour in a direction at right angles to the length of the bar.

(V. 1894.)

227. The axles of the carriages of a train travelling at 60 miles an hour are 5 ft. 3 in. long. Find the difference of potential between their ends, given 1 ft. = 30.5 cms., and the vertical component of the earth's field = 0.41 c.g.s. unit.

(V. 1898.)

228. A river 100 metres broad is flowing northwards along an insulated bed at the rate of 2 metres per sec.; calculate the difference of potential between the water at the two sides, given that $H = 0.17$ c.g.s. unit and that the magnetic dip is 60° . Does the direction of flow make any difference? (V. 1896.)

229. A copper disc of 10 cms. radius spins on its axis (perpendicular to its plane) 3000 times a minute in the earth's horizontal field of force; find the E.M.F. between the centre and the circumference of the disc. ($H = 0.18$.) (V. 1890.)

230. A disc of diameter 30 cms. rotates 20 times a sec. in the field due to a very long solenoid of 10 turns per cm. through which 1 ampere is flowing. Find the potential difference between the centre and the circumference of the disc.

231. A circular metal disc of radius 30 cms. rotates on its axis 70 times a second in a uniform magnetic field of intensity 400 c.g.s. units, the disc being normal to the lines of force; calculate in amperes the current generated when a fixed wire touches the disc at the centre and at the circumference so as to complete a circuit of one ohm electrical resistance. (V. 1897.)

232. A solid fly-wheel 1 metre in diameter spins on an axis which is directed towards the North at a place where the horizontal intensity is 0.18. It makes 250 revolutions per min.; calculate the difference of potential between the centre and the circumference of the wheel. (L. 1895.)

233. Find the quantity of electricity discharged through a circuit containing a small coil of n turns, when the coil is slipped off from the centre of a long bar magnet of pole-strength μ , the resistance of the whole circuit being R ohms. (V. 1903.)

234. Find the quantity of electricity induced when a coil of 20 turns, 15 cms. radius, and 4 ohms resistance, lying on a table, is turned over in the earth's magnetic field, the angle of dip being 60° and the value of H being 0.18 c.g.s. unit.

235. Find the total quantity of electricity sent through a galvanometer connected to a coil of ten windings, when the coil is rotated through a right angle about a vertical axis, its final position being in the magnetic meridian. Mean area of coil, 1000 square cms.; resistance of coil and galvanometer, 1 ohm; horizontal force of earth's magnetism, 0.17 dyne per unit pole. (V. 1902.)

236. A flat coil of wire of n turns of mean radius a and mass m has its ends joined and rests on a horizontal table. The density and specific resistance of the material are ρ and σ respectively. Calculate an expression for the quantity of electricity induced when the coil is turned over, given that the horizontal component of the earth's field is H and the dip d . (V. 1905.)

237. A circular coil 12 cms. in radius and consisting of 150 turns is rotated about a vertical diameter at the rate of 4 revolutions per second. Taking the horizontal component of the earth's magnetic force to be 18 c.g.s. unit, find the average electromotive force induced in the coil, and also the maximum value of the electromotive force. (V. 1901.)

238. A wire in the form of a circle 10 cms. in diameter is spinning about a diameter as axis (which is placed vertically) at the rate of 10 revolutions per second in the earth's magnetic field ($H = 0.18$). Calculate the maximum and average E.M.F. generated during a revolution. (V. 1894.)

239. A straight metal bar of length l is suspended horizontally in the magnetic meridian from a beam by means of

two light vertical wires of length L , attached to its extremities, and is set in pendular oscillation in a plane perpendicular to its length, at a region where the earth's field is uniform. Find the magnitude of the E.M.F. generated in the rod and suspending wires at any instant.
(V. H. 1902.)

240. A cylindrical tube 50 cms. long and 2 cms. in diameter is uniformly wound with 2000 turns of wire. Calculate the self-induction.
(V. 1903.)

241. Calculate roughly the self-induction of 160 metres of wire wound uniformly in the form of a straight solenoid 80 cms. long and 3 cms. in diameter.
(V. 1905.)

242. If a wooden curtain ring of mean diameter 21 cms. and sectional diameter 3 cms. be uniformly wound with 1000 turns of very fine wire, calculate its coefficient of self-induction.
(L. H. 1891.)

243. Find the resistance which will be just sufficient to prevent the discharge of a 40 microfarad condenser through a self-induction of half a henry from being oscillatory.
(V. H. 1896.)

244. A condenser is formed by means of two plates having an area of 1 sq. metre at a distance of 1 mm.; it is discharged through a coil having a length of 1 metre and a mean area of 1 sq. cm.; the resistance of the coil is 1000 ohms. Calculate roughly the number of turns in the coil, such that the discharge may just not be oscillatory.
(V. H. 1893.)

245. By means of the formula $E = a(t_1 - t_2) + b(t_1^2 - t_2^2)$, shew that the E.M.F. in a thermoelectric circuit may also be represented as the difference of two terms, each of which involves only one of the temperatures t_1 and t_2 together with the neutral temperature.
(V. 1899.)

246. A thermoelectric circuit consists of two equal bars of copper and iron with the ends joined and the junctions maintained respectively at 19.5° and 20.5° C. Compare the amounts of heat converted into electromagnetic energy and conducted along the bars, given that the thermoelectric power at 20° C. = 18 microvolts, the specific resistances of copper and iron are 1.6 and 9.8 microhms, and the heat conductivities of the two are 1 and 0.2 c.g.s. unit respectively. (L. H. 1894.)

247. The thermoelectric power of a copper-lead circuit in microvolts is $-1.34 - 0.0094t$ at temp. t , and for an iron-lead circuit is $-17.2 + 0.048t$. Calculate the E.M.F. in a copper-iron circuit whose junctions are at 50° and 250° C. respectively. (V. H. 1896.)

248. The velocity of an ion through air at atmospheric pressure under a potential gradient of 1 volt per cm. is about 1.5 cm. per sec.; assuming that the frictional resistance varies as the velocity, and that the ionic charge is 10^{-19} coulomb, how fast would it move under a force of 1 dyne?

249. Given that the velocity of a hydrogen ion in electrolysis is 320×10^{-5} cm. per sec. for a potential gradient of 1 volt per cm., find the force necessary to keep 1 gm. of hydrogen moving under similar conditions at a speed of 1 cm. per sec., if an ampere liberates 0.0001035 gm. per sec.

250. Obtain the corresponding result for 1 gm. of potassium (atomic weight = 39), given that the velocity of the ion is 66×10^{-5} cm. per sec. for a gradient of 1 volt per cm. (N.B. The forces determined in this and the previous example may be called "ionic friction coefficients," and the student should notice the enormous frictional resistance.)

251. Define the specific molecular conductivity for electricity of a given salt, and shew how it is connected with

the sum of the ionic velocities. If a solution of 1 gram-equivalent in 50 litres has conductivity $\cdot 002244$, what is the specific molecular conductivity?

252. In an experiment on the electrolysis of silver nitrate, 1.26 gm. of silver is deposited; also a titration of the solution round the kathode gave 17.46 gms. of silver chloride before, and 16.68 gms. after, the electrolysis; if the atomic weights of silver and chlorine are 108 and 35.5 respectively, determine the ratio of the velocities of the two ions of silver nitrate.

253. Find in electrostatic units the charge of electricity of each kind associated with 1 c.cm. of hydrogen at N.T.P., given that a current of 1 ampere passing through acidulated water decomposes $\cdot 00009315$ gm. of water per sec., and that a litre of hydrogen at N.T.P. weighs $\cdot 09$ gm.

254. Determine the radius of the drops of water forming a cloud from the fact that it is observed by means of a microscope to fall at the rate of 0.12 cm. per sec., given that the velocity of fall is $\frac{2}{9}a^2g\rho/\mu$ where a is the radius and ρ the density of the drop, and μ is the viscosity of the air in which the cloud falls, viz., $\cdot 00018$ c.g.s. unit.

255. Ionised air under atmospheric pressure, and saturated with moisture at 16° C., is suddenly expanded; find the lowest temperature reached before cloud-formation begins, given that the height of the barometer is 773 mms., that the saturated vapour pressure of water at 16° is 13 mms., and that the ratio of the specific heats of air is 1.41; also that when the air is saturated at its new volume and 16° , the pressure is 572 mms.

256. Condensation ensues upon the sudden expansion described in the previous question, and proceeds until the air is again only just saturated; shew that the temperature at which this state of things is reached is $1^\circ 2$ C., given that the

density of saturated aqueous vapour at 16° is 135×10^{-7} and at $1^\circ 2$ is 51.3×10^{-7} , while the latent heat of condensation is 606, the specific heat of air at constant volume is .167, and the mass of 1 c.cm. of air at N.T.P. is .001293 gm. (The capacity for heat of the aqueous vapour is negligible.)

257. In connection with Qns. 255 and 256, determine the weight of the water-globules in 1 c.cm. of the cloud formed.

258. Assuming that Qns. 254 to 257 relate to the same experiment, and that there are as many water-globules as ions of both kinds, determine the number of ions in 1 c.cm. of air before expansion.

259. By the ionisation of the air in the experiment of Qns. 254 to 258, electricity was caused to leak off a zinc plate 3.2 cms. in diameter; a quadrant electrometer connected to the plate showed originally a deflection of 500 divisions, which the leakage reduced by 57 divisions per minute; the space across which the leakage took place had a uniform thickness of 1.2 cm., and the capacity of the zinc plate and connected quadrants was 62 electrostatic units. From these data and those of Qns. 254 to 258 and from Rutherford's value of the mean ionic velocity in air at N.T.P., viz. 1.5 cm. per sec. for a potential gradient of 1 volt per cm., calculate the charge on an ion.

260. Shew that if a conducting sphere of radius a carries an electric charge e and is surrounded by a dielectric of specific inductive capacity K , there is a repulsion on the surface of $\frac{e^2}{8\pi K a^4}$ per unit area.

261. A soap-bubble of radius a and surface-tension T receives an electric charge e . Assuming that the air inside

the bubble obeys Boyle's Law, shew that the new value of the radius x is to be calculated from the equation

$$\left(P + \frac{4T}{a}\right) a^3 = \left(P + \frac{4T}{x} - \frac{e^2}{8\pi K x^4}\right) x^3,$$

where P and K denote the pressure and specific inductive capacity of the gas outside the bubble.

262. Shew how the equation of the preceding example must be modified if at the time of charging the soap-bubble it is contained in a rigid concentric spherical envelope of radius r .

263. Find the numerical solution to Q. 261 in the case in which the original radius is 1 cm., the surface tension is 75 dynes per cm., the pressure of the external air is 1000 dynes per sq. cm., and the electric charge is given by connecting the bubble with a source at 30,000 volts.

264. Shew that if the sphere of Q. 57 is electrified with a charge e , the equation becomes

$$P \log_e (P'/P) = \frac{\rho}{\sigma} \left(P' - P + \frac{2T}{a} - \frac{e^2}{8\pi K a^4} \right).$$

265. Find the effect of electric charge on the difference of pressure inside and outside a water-globule of radius .001 mm., carrying a charge sufficient to raise it to a potential of 4 volts, the surface tension being 75 dynes per cm.

266. What electric charge would neutralise the effect of surface tension in the case of a water-globule of radius (a) 1 cm., (b) .001 mm., the tension of a surface separating air and water being 75 dynes per cm.? State the result in electrostatic units and also in coulombs in each case. To what potential would the globules be raised respectively?

267. Find the radius of a water-globule of such a size that the quantity of electricity carried by a gaseous ion (about 3.4×10^{-10} electrostatic units, according to J. J. Thomson) would, if placed upon the globule, exactly neutralise the effect of surface-tension upon the vapour pressure outside it, assuming the surface-tension to be 75 dynes per cm. as for large drops. (Note that the result is of the same order as the "radius of molecular action.")

268. Supposing the quantity of aqueous vapour in the air to be $1/\epsilon$ times that which would produce saturation over a flat water-surface, find from the equation of Q. 264 the size of the drops which would be in equilibrium in such an atmosphere, if each drop carried a charge of 3.4×10^{-10} electrostatic units. (Temp. 15°C.)

269. Find to what extent the air in Q. 268 must be dried in order that the drops which are in equilibrium may be half as large as those which would be so over a flat surface.

270. Shew from the equation of Q. 264 that if globules of water carry a charge of 3.4×10^{-10} electrostatic units, then globules of two different sizes will be in equilibrium with any given vapour pressure below a certain limiting value. Find the ratio of this limiting value to that which applies outside a flat surface. Find also the ratio of the corresponding radius to the radius of the globules of Q. 267.

271. Shew from the equation of Q. 264 that if a very small water-globule carry a charge of 3.4×10^{-10} electrostatic units, it cannot grow to a large size unless the air is considerably supersaturated. Determine the amount of supersaturation necessary.

(N.B. In Qns. 265 to 271, the saturated vapour is assumed to obey Boyle's law, and the surface tension to be the same as for large drops.)

272. Draw a graph to illustrate Q. 264 shewing how P'/P varies with the radius of the drop, the charge on it being supposed constant. Explain the significance of the form of the graph, especially of the existence of a maximum on it.

273. Shew the bearing of the facts illustrated in the preceding questions on the phenomena of cloud formation; also on our conception of the size of molecules.

274. Assuming that the charge on an atom of hydrogen in electrolysis is the same as that on a gaseous ion, say 3.4×10^{-10} electrostatic units, determine the number of molecules in 1 c.cm. of the gas at N.T.P., given that 1 litre weighs .09 gm., and that a coulomb liberates .00001035 gm.

ANSWERS.

1. 746 watts.
4. $x = \frac{gt}{k} + \frac{(\epsilon^{-kt} - 1)}{k} \left(\frac{g}{k} - V \right)$, where x and t are the distance and time reckoned from the position where the velocity was V . Terminal velocity, $\frac{g}{k}$.
8. H.P.=38190; time=127.6 hrs.; resistance=236.24 ton weight; coal=4353 tons. (*Geographical* miles are meant.)
9. 6.81×10^{-12} kgm.-weight. 10. 3762 secs.
11. Mass of sun : mass of earth = $(390)^3 : (13)^3$.
12. 11.2 lbs. per ton; 358.4 H.P.
13. 4400 ft.-lb.-sec. units per sec.; 17600 ft.-poundals per sec.
14. $140.15 : g$. 15. $\frac{2\pi^2}{g}$. 16. $\frac{3}{4}$.
17. $\frac{1.00003}{2.70} g$ cms. per sec.
18. $\sqrt{480/g}$ secs.; $\sqrt{15g/2}$ radians per sec.
19. $96\frac{3}{17}$ cms. per sec. per sec.
20. If the mass M be suspended at an arm r , and the moment of inertia of the wheel and axle be I , the acceleration = $\frac{Mgr}{Mr^2 + I}$. If a length l unwrap in time t , the friction couple is

$$Mgr - \left(Mr + \frac{I}{r} \right) \frac{2l}{t^2}.$$
21. $\sqrt{\frac{3g}{a}}$.
22. When the rod has turned through an angle θ , the two amounts are $\frac{1}{8} mgl \sin \theta$ and $\frac{3}{8} mgl \sin \theta$ respectively.

23. Let the added mass have a radius of gyration k , and let its centre of gravity be x below the knife-edge; then the positions are given by $\frac{k^2 + x^2}{x} < = > \frac{g}{\pi^2}$ respectively. 25. (i) $a \cdot \frac{\sqrt{3}}{2}$; (ii) $a \cdot \frac{5\sqrt{3}}{12}$.
26. (i) $a \cdot \frac{2\sqrt{2}}{3}$ below the point of suspension; (ii) $\frac{5a}{6}$ below.
27. Becomes $\sqrt{1.2}$ times as long. 28. 52.88 cms.
29. $2\pi \sqrt{\frac{D}{g}}$.
30. (i) Multiplied by $4\sqrt{2}$; (ii) divided by $2\sqrt{2}$; (iii) doubled.
31. $\frac{27}{160}$ sec. 32. 2.8×10^7 pounds' weight per sq. in.
33. $2.5 \times 10^9 g$ dynes per sq. cm. 34. Halved.
35. $\frac{9\pi g}{25600}$ cm. 36. 0.1875 cm. 37. $\frac{10000}{3} g$ ergs per c. cm.
38. Torsional rigidity, about $2.23 \times 10^{10} g$ dynes per sq. cm.; simple rigidity, $2/\pi$ times as much.
39. Five per cent., if the errors in n and Y are in opposite senses.
40. 22001 : 22010.7. 41. 0.000474 gm.
42. $\frac{10000}{7g}$ cms., assuming that the liquid wets the tube.
43. 2.88×10^8 ; $\frac{2880}{g}$ cms. 44. 1.2 cm.
45. 296000 dynes per sq. cm. 46. 0.0809 cm.
47. 0.1764 poundal per ft. 48. $\frac{3g}{40}$ dynes per cm. 49. $\frac{2TA}{x}$.
50. $(76s + 10)g + 156000$, where s is the density of mercury.
51. 125960 ergs. 52. $\frac{1}{20}$ gm. weight per sq. cm. 53. $\frac{36}{17g}$ cm.
54. $\frac{P' + 24}{0.2214P}$, where P' is the external pressure and P is a normal atmosphere, both in dynes per sq. cm.
58. (a) 0.0192, 1.016; (b) 19.2, 1016 dynes per sq. cm.
60. (a) 1.152×10^{-7} cm., better 1.662×10^{-7} ; (b) 1.016×10^{-6} , better 1.466×10^{-6} .
61. 0.00077 cm. 62. 226 cms.

63. Let the pressure outside the tyre be one atmosphere, and let the internal pressure be n atmospheres; then the time taken is about $863000 (n^2 - 1) \log_{10} \left(\frac{n^2 + n - 2}{n^2 - n - 2} \right)$ seconds. If $n=2$, this is infinite (as it should be); if $n=3$, the time is 2.75×10^6 secs.; for $n=4$, 3.30×10^6 secs.; for $n=10$, 7.60×10^6 ; for $n=20$, 15.05×10^6 . When the internal pressure considerably exceeds the external (as was probably intended), a rougher calculation is appropriate, and leads to the result $750000 n$ secs. It will be noticed that at 20 atmospheres, this is practically correct.
64. 1:17. 65. Observer at rest, 19:21; observer on train, 361:441.
66. $1086\frac{2}{3}$; 1080. 67. 1.34×10^{12} dynes per sq. cm.
68. 4.032×10^{11} dynes per sq. cm.
69. $2^{19} \times 17000$ poundals per sq. ft.
70. $10^9 g$ dynes per sq. cm.; $\sqrt{\frac{10^9 g}{3}}$ cms. per sec.
71. Frequency $\frac{1}{2} \sqrt{\frac{1100 g}{7}}$.
72. $\frac{10}{3} \sqrt{\frac{50 g}{77 \pi}}$. 74. 120, taking $g=980$ cms. per sec. per sec.
75. $\frac{101000}{3}$ cms. per sec. 76. 33047 cms. per sec.
77. $35\frac{5}{12}$ cms. 78. $\frac{125}{2592}$ c. cm. 79. $-\frac{1}{10000}$.
80. The temperature-coefficient = -2.11×10^{-4} .
81. 1.0011; .9989. 82. 75.604 cms.
83. 1 metre = 1.09376 yd. 84. 475×10^{-5} . 85. 30.02278 gms.
86. $\frac{23}{180}$ of the internal volume should be filled.
87. Force exerted per degree = coefficient of expansion multiplied by force necessary to double the length of the bar (upon the usual understanding).
88. Gains 5.646 secs. per day for each degree rise of temperature.
89. About 10.2 secs. 90. About 288°C . 91. 0.0536.
92. 1.402. 93. 0.2384. 94. 6000.
95. 0.00001259. 96. 1:0.365:4.34. 97. 8:1; equal.
98. (i) 3:1; (ii) 81:16. 99. $21^\circ 15'$. 100. 1.194 gm.
101. 13°C . 102. 30.55, 49.71, 81.86 cms. respectively.

103. 0.3424. 104. $100 \sqrt{15sg}$, where s is the density of mercury.
105. 46612 cms. per sec. 106. 8.5×10^{-6} cm.
107. If θ is the original absolute temperature, then the new temperature is the antilog. of $\{\log \theta + (\gamma - 1) \log 10\}$, where γ is the ratio of the specific heats.
108. (i) 467°C. ; (ii) $288^\circ.5 \text{C.}$, taking $\gamma = 1.41$.
109. (a) 50 atmospheres, (b) 239 atmospheres; (a) no rise, (b) 1032°C.
110. $-125^\circ.6 \text{C.}$, taking $\gamma = 1.41$. 111. 2184°C.
112. $\frac{5}{83}$ calories per gm. 113. 2.02×10^9 ergs per gm.
114. JL/g , where J is Joule's equivalent, L the latent heat of fusion, and g the acceleration of gravity.
115. About 26300 cms. per sec.
116. $0^\circ.885 \text{C.}$, but the question is faulty, the data being inconsistent.
117. Internal, 20.8×10^9 ergs; external, 1.71×10^9 .
118. 0.0684. 119. About 2.99 times as much as is needed.
120. Gain of 0.00368 units. 121. Loss of 1.75 units.
122. $\frac{6.95 \times 10^7}{JL}$ lbs., J being in ft.-lbs. per calorie.
123. (b) is $12\frac{2}{3}$ times as good as (a). 124. 1672 c. cms.
125. Falls $0^\circ.3755 \text{C.}$ 126. $0^\circ.0252 \text{C.}$ 127. $-\frac{1}{23880}$.
128. Let c be the capacity for heat (in ergs) of the substance of the bubble, θ its initial (absolute) temperature, and A the increase in area; then if surface tension varies uniformly with temperature, the difference in work is $\theta(0.17A - c)$ very approximately.
129. Mean interval ± 15.29 secs. 130. 48 hrs. ± 17.21 secs.
131. 2 kilometres per sec.
132. Away from the earth with $\frac{2}{1388}$ of the velocity of light.
133. 25 cms. below the true centre.
134. For a Huyghens eye-piece, focal lengths 2 ins. and $\frac{2}{3}$ in.; for a Ramsden eye-piece, two lenses each of focal length $\frac{4}{3}$ in.
135. (a) 2 or $\frac{1}{2}$; (b) 2 or $\frac{1}{2}$; (c) 200 or $\frac{1}{200}$.
136. (i) 1 : 100 : 576, (ii) 1 : 1 : 0.0576. The higher power.
137. The first. $\frac{2100}{79}$ cms. convex, $\frac{2100}{37}$ cms. concave.
138. 10.85 ins. convex, 17.00 ins. concave. 139. $p_1 + p_2 + p_1 p_2 d$.

140. 20 cms.; infinity. 141. $18\frac{3}{4}$ ins. 142. 1.5.
143. 6 ins. 144. $2\frac{7}{12}$ ins.
145. $\frac{4.0}{14.9}$ cm. and $\frac{6.0}{14.9}$ cm. from faces.
146. (i) $3\frac{1}{4}$ ins.; $2\frac{7}{12}$, $3\frac{1}{4}$. (ii) $2\frac{1.9}{2.8}$ ins.; $2\frac{9}{2.8}$, $2\frac{9}{2.8}$. (iii) $5\frac{5}{8}$ ins.; $6\frac{3}{8}$, $4\frac{3}{8}$.
147. 3.6 ins.; 3 ins. and 4 ins.; 1.5.
148. (i) Water against flat surface; nodal points at $1\frac{1}{12}$ and $\frac{7}{12}$ inch from curved and flat surfaces; focal points at $3\frac{1}{4}$ and $3\frac{3}{8}$ ins. from the same. (ii) Water against curved surface; the corresponding values are $3\frac{1}{4}$, $3\frac{11}{12}$, 13, and $9\frac{1}{12}$.
149. (i) $8\frac{5.6}{8}$ or $8\frac{3.5}{8}$ ins. according as the object is on the side of the air or the water. (ii) The corresponding values are $27\frac{4}{87}$ and $8\frac{1}{47}$ ins. Magnification: (i) $\frac{1.3}{11}$ or $\frac{3.9}{23}$; (ii) $\frac{1.7}{7}$ or $\frac{1.17}{37}$.
150. 0.0398 sq. cms.
151. Nodal points at $\frac{3.5}{8}$ and $\frac{3.5}{8}$ inch from the faces in contact with air and water respectively; focal points at $3\frac{41}{88}$ and $4\frac{11}{16}$ ins.
152. (i) $13\frac{31}{133}$ ins. (ii) $16\frac{41}{144}$ ins. Magnification: (i) $\frac{3.25}{33}$; (ii) $\frac{7.5}{22}$.
153. $\frac{1}{100}$ inch; 7.5 ins. 154. 8 metres.
155. $\frac{1}{12000}$ of a radian. 156. 0.0135 cm. 157. 0.0295 cm.
158. $\frac{0.001525}{\lambda}$ cm. 159. 85.2 cms. 160. 5×10^{-5} cm.
161. 3.535×10^{-5} cm. 162. 2.654×10^{-5} cm.
163. 1.01836 mm. 164. $\frac{2.9}{6000}$. 165. 108.
166. At all points on two surfaces of revolution having an asymptotic cone of semi-vertical angle $54\frac{3}{4}^\circ$ with its apex at the middle point of the magnet.
167. (i) The two points on the horizontal section of the above surfaces where the force due to the magnet is equal and opposite to H . (ii) On the meridian, at a distance r from the centre given by the equation $2Mr = H(l^2 - r^2)^2$; where M and $2l$ denote the moment and length of the magnet.
168. 1157 c.g.s. units. 169. 0.98. 170. 0.0468 per cent.
171. If the magnetic moments are M and M' , and the distance between the centres is d , the couples are $\frac{MM'}{d^3}$ and $\frac{2MM'}{d^3}$, both in the same sense.
172. 109 : 416.

173. If the length of the line joining the centres is d , the field is either $8\sqrt{2} \cdot \frac{M}{d^3}$ at right angles to that line, or $16\sqrt{2} \cdot \frac{M}{d^3}$ along it.
174. If d is the distance between the centres, the field is $8\sqrt{5} \frac{M}{d^3}$ in a direction inclined to the line of centres at $\cot^{-1} 2$.
175. If the magnets are at distances a and b from the point, the field is $\frac{2M\sqrt{a^6+b^6}}{a^3b^3}$ inclined at an angle $\tan^{-1} \left(\frac{a^3}{b^3} \right)$ to the distance a .
176. $\frac{M}{a^3} (13.5 \pm 6\sqrt{3})$ along the direction of the third magnet.
177. (i) $\frac{3\sqrt{3}}{2} \cdot \frac{M}{a^3}$ along the median, or $\frac{1}{2} \frac{M}{a^3}$ at right angles to it;
 (ii) $\frac{27}{2} \frac{M}{a^3}$ along, or $\frac{15\sqrt{3}}{2} \frac{M}{a^3}$ at right angles.
178. (i) $2\sqrt{3} \cdot \frac{M}{a^3}$ along the median or $\frac{2M}{a^3}$ at right angles to it;
 (ii) $\frac{9}{2} \frac{M}{a^3}$ along, or $\frac{21\sqrt{3}}{2} \frac{M}{a^3}$ at right angles.
179. $\frac{2}{5} \sqrt{\frac{g}{5}}$ electrostatic units.
180. $\frac{A}{4\pi d}$; $\frac{1}{d}$.
181. $\sqrt{\frac{g}{20\pi}}$ c.g.s. units.
182. 44.2 ergs.
183. 451 grams' weight.
184. 100890 volts, taking an atmosphere as 10^6 dynes per sq. cm.
185. 10.8 cms.; $3\frac{1}{2}$ units.
186. If Q = the charge on the particle and Q' = that on the sphere of radius a , then the limiting distance d is given by $\frac{d^2(d+2a)^2}{a(d+a)^3} = \frac{Q}{Q'}$.
187. 8.70×10^7 ohms.
188. (a) $\frac{25}{3 \times 10^9}$, (b) $\frac{25}{3 \times 10^8}$, (c) $\frac{25}{3 \times 10^7}$.
189. $\frac{1}{36\pi}$.
190. 10.
191. 0.829.
192. Electrical : coal = 3730 : 99.
193. 36×10^{12} ergs; 10 amperes; 2 volts.
194. 4.91×10^{-11} penny; 20357 kilometres.
195. 0.294 watt; 1.764 volt.
196. 4160 watts; 2048 watts.

197. 71.22 ohms. 198. 0.318 units.
199. $10J$ absolute units. 200. $0.913 \frac{p}{V}$ pence.
201. 0.3402 gm.; 142.3 calories, 69.7 calories, 46.5 calories.
202. 1.508 volt. 203. $18\frac{2}{3}$ years.
204. 8.70×10^6 volts (quite impracticable, of course).
205. 1.4862; 20 calories per sec.
206. (a) 38.1, (b) 8.48 calories per sec.
207. $\delta_A : \delta_B = 1 : n^3$, the two needles being magnetised with equal intensity.
208. $2\pi nC \left\{ 1 - \frac{x}{\sqrt{r^2 + x^2}} \right\}; \frac{2\pi nCr^2m}{(r^2 + x^2)^{\frac{3}{2}}}; x = \frac{r}{2}; 0.2662$ units.
209. $\frac{1}{81} c.g.s.$ units. 210. 0.002066 calories per sec.; 0.006198.
211. π c.g.s. units.
212. $\frac{8.992}{r}$, where r = the radius of either coil.
213. $\frac{2}{3}$ amperes. 214. $\pi a^2 i H$. 215. 3.72×10^8 cms. per sec.
216. 3.085×10^{10} cms. per sec. 217. $\frac{4}{3}$ ohms.
218. A piece of the same wire of length $(2 - \sqrt{2})$ times the side of the square.
219. $\frac{1}{31}$ ampere or $\frac{1}{3}$ ampere.
220. Let R be the resistance of each arm, and E be the E.M.F. of each battery; then the two batteries and two opposite arms each transmit a current $\frac{1}{2} \frac{E}{R}$, while the other two arms carry no current.
221. (a) $\frac{11}{10}$ amp. in each case. (b) Open, $\frac{9}{10}$ amp.; closed, $\frac{9}{10}$ amp.
222. $\frac{11}{10}$ and $\frac{9}{10}$ amp. respectively.
223. $\frac{3}{11}$ amp. in the larger battery, $\frac{1}{11}$ in the smaller, $\frac{1}{11}$ in the wire.
224. 0.72, 0.464, 0.256 amp. respectively.
225. $\frac{2}{15}$ amp., $\frac{3}{2}$ amp., $\frac{41}{310}$ amp. respectively.
226. 0.00193 volt. 227. 0.00176 volt.
228. 0.00589 volt. 229. $\frac{9\pi}{10^6}$ volt. 230. $\frac{18\pi^2}{10^5}$ volt.
231. 0.792 ampere. 232. 5.89×10^{-5} volt.

233. $\frac{4\pi\mu n}{R} \times 10^{-8}$ coulomb.

234. 2.204×10^7

235. 1.7×10^{-5} coulomb.

236. $\frac{mH \tan d}{2\pi n \sigma \rho}$.

237. 0.001954 volt; 0.00307 volt.

238. $\frac{9\pi^2}{10^7}$ volt; $\frac{18\pi}{10^7}$ volt.

239. When the wires are inclined at θ to the vertical, the E.M.F. is $2lV \cos \theta \sqrt{gL \left(\sin^2 \frac{\alpha}{2} - \sin^2 \frac{\theta}{2} \right)}$, where V is the vertical component of the earth's field, and α is the maximum angular displacement.

240. 0.00316 henry.

241. 32×10^{-4} henry.

242. 0.001346 henry.

243. 223.6 ohms.

244. 4194 turns.

246. 9 : 1596000.

247. 0.00145 volt.

248. 1.5×10^{12} cms. per sec.

249. 3.02×10^{14} dynes.

250. 3.85×10^7 kilograms' weight.

251. Sum of velocities = $EV_x \times \text{sp. mol. cond.}$, where E is the electrochemical equivalent of hydrogen, and V_x is the potential gradient.

$112.2 \frac{\text{sq. cms.}}{\text{ohm gm.}}$

252. 1 : 1.146.

253. 1.3×10^{10} units.

254. 3.15×10^{-4} cm.

255. $-18^\circ 2$ C.

257. 4.80×10^{-6} gm.

258. 5.0×10^4 .

259. 7.8×10^{-10} electrostatic units.

263. 1.111 cm.

265. Decreases it by $\frac{1}{675\pi}$ of itself.

266. (a) 61.4 units, 2.05×10^{-8} coulomb; 18420 volts. (b) 6.14×10^{-5} units, 2.05×10^{-14} coulomb; 184.2 volts.

267. 3.13×10^{-8} cm.

268. 2.90×10^{-8} cm.

269. The vapour pressure must be reduced to 8.6×10^{-23} times the saturation-pressure over the flat surface.

270. 5.55 : 1; 1.587 : 1.

271. At least 5.55 times the normal saturation-pressure.

274. About 3.84×10^{19} .

